General Certificate of Education
June 2009
Advanced Subsidiary Examination

MATHEMATICS **MPC1**
Unit Pure Core 1

Wednesday 20 May 2009 1.30 pm to 3.00 pm

For this paper you must have: • an 8-page answer book • the blue AQA booklet of formulae and statistical tables. You must **not** use a calculator.	

Time allowed: 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MPC1.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.
- The use of calculators (scientific and graphics) is **not** permitted.

Information
- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

Answer **all** questions.

1 The line AB has equation $3x + 5y = 11$.

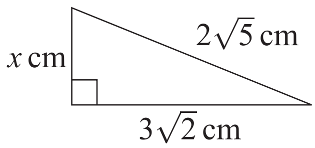
(a) (i) Find the gradient of AB. *(2 marks)*

(ii) The point A has coordinates $(2, 1)$. Find an equation of the line which passes through the point A and which is perpendicular to AB. *(3 marks)*

(b) The line AB intersects the line with equation $2x + 3y = 8$ at the point C. Find the coordinates of C. *(3 marks)*

2 (a) Express $\dfrac{5+\sqrt{7}}{3-\sqrt{7}}$ in the form $m + n\sqrt{7}$, where m and n are integers. *(4 marks)*

(b) The diagram shows a right-angled triangle.



The hypotenuse has length $2\sqrt{5}$ cm. The other two sides have lengths $3\sqrt{2}$ cm and x cm. Find the value of x. *(3 marks)*

3 The curve with equation $y = x^5 + 20x^2 - 8$ passes through the point P, where $x = -2$.

(a) Find $\dfrac{dy}{dx}$. *(3 marks)*

(b) Verify that the point P is a stationary point of the curve. *(2 marks)*

(c) (i) Find the value of $\dfrac{d^2y}{dx^2}$ at the point P. *(3 marks)*

(ii) Hence, or otherwise, determine whether P is a maximum point or a minimum point. *(1 mark)*

(d) Find an equation of the tangent to the curve at the point where $x = 1$. *(4 marks)*

4 (a) The polynomial $\mathrm{p}(x)$ is given by $\mathrm{p}(x) = x^3 - x + 6$.

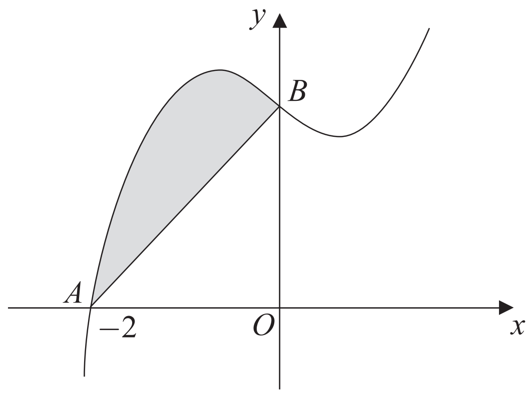
(i) Find the remainder when $\mathrm{p}(x)$ is divided by $x - 3$. *(2 marks)*

(ii) Use the Factor Theorem to show that $x + 2$ is a factor of $\mathrm{p}(x)$. *(2 marks)*

(iii) Express $\mathrm{p}(x) = x^3 - x + 6$ in the form $(x + 2)(x^2 + bx + c)$, where b and c are integers. *(2 marks)*

(iv) The equation $\mathrm{p}(x) = 0$ has one root equal to -2. Show that the equation has no other real roots. *(2 marks)*

(b) The curve with equation $y = x^3 - x + 6$ is sketched below.



The curve cuts the x-axis at the point $A\,(-2, 0)$ and the y-axis at the point B.

(i) State the y-coordinate of the point B. *(1 mark)*

(ii) Find $\displaystyle\int_{-2}^{0} (x^3 - x + 6)\,\mathrm{d}x$. *(5 marks)*

(iii) Hence find the area of the shaded region bounded by the curve $y = x^3 - x + 6$ and the line AB. *(3 marks)*

Turn over for the next question

5 A circle with centre C has equation

$$(x-5)^2 + (y+12)^2 = 169$$

(a) Write down:

(i) the coordinates of C; *(1 mark)*

(ii) the radius of the circle. *(1 mark)*

(b) (i) Verify that the circle passes through the origin O. *(1 mark)*

(ii) Given that the circle also passes through the points $(10, 0)$ and $(0, p)$, sketch the circle and find the value of p. *(3 marks)*

(c) The point $A(-7, -7)$ lies on the circle.

(i) Find the gradient of AC. *(2 marks)*

(ii) Hence find an equation of the tangent to the circle at the point A, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers. *(3 marks)*

6 (a) (i) Express $x^2 - 8x + 17$ in the form $(x-p)^2 + q$, where p and q are integers. *(2 marks)*

(ii) Hence write down the minimum value of $x^2 - 8x + 17$. *(1 mark)*

(iii) State the value of x for which the minimum value of $x^2 - 8x + 17$ occurs. *(1 mark)*

(b) The point A has coordinates $(5, 4)$ and the point B has coordinates $(x, 7-x)$.

(i) Expand $(x-5)^2$. *(1 mark)*

(ii) Show that $AB^2 = 2(x^2 - 8x + 17)$. *(3 marks)*

(iii) Use your results from part (a) to find the minimum value of the distance AB as x varies. *(2 marks)*

7 The curve C has equation $y = k(x^2 + 3)$, where k is a constant.

The line L has equation $y = 2x + 2$.

(a) Show that the x-coordinates of any points of intersection of the curve C with the line L satisfy the equation

$$kx^2 - 2x + 3k - 2 = 0$$ *(1 mark)*

(b) The curve C and the line L intersect in two distinct points.

(i) Show that

$$3k^2 - 2k - 1 < 0$$ *(4 marks)*

(ii) Hence find the possible values of k. *(4 marks)*

END OF QUESTIONS

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